Received: *June 11, 1999*, Accepted: *June 30, 1999*

Ramond-Ramond couplings of non-BPS D-branes*

Marco Billò

Dipartimento di Fisica Teorica, Università di Torino and
I.N.F.N., Sezione di Torino, via P. Giuria 1, I-10125, Torino, Italy, and
Instituut voor Theoretische Fysica
Katholieke Universiteit Leuven, B-3001 Leuven, Belgium
E-mail: billo@to.infn.it

Ben Craps† and Frederik Roose

Instituut voor Theoretische Fysica
Katholieke Universiteit Leuven, B-3001 Leuven, Belgium
E-mail: Ben.Craps@fys.kuleuven.ac.be,
Frederik.Roose@fys.kuleuven.ac.be

Abstract: We study how non-BPS type II D-branes couple to R-R potentials. Upon tachyon condensation the couplings we find give rise to the Wess-Zumino action of BPS D-branes.

Keywords: Superstrings and Heterotic Strings, D-branes.

*Work supported by the European Commission TMR programme ERBFMRX-CT96-0045.
†Aspirant FWO, Belgium

JHEP06(1999)033

In a seminal series of papers, Sen has shown how to obtain an exact conformal field theory description of non-BPS D-branes (see [1] for two of the main papers and two recent reviews). In this setup, a non-BPS D$(p-1)$-brane remains after tachyon condensation on a Dp–anti-Dp-brane pair. In type II theories the resulting object still has a tachyonic mode on its worldvolume, but performing the orientifold to type I stabilizes the non-BPS brane. In a related development, D-brane charges have been shown to take values in appropriate K-theory groups of space-time. A major result is that all lower-dimensional D-branes can be considered in a unifying manner as non-trivial excitations on the appropriate configuration of higher-dimensional branes. For type IIB this was demonstrated by Witten in [2], where all branes are built from sufficiently many D9–anti-D9 pairs. As to type IIA, Horava outlined the construction of lower branes from non-BPS D9-branes [3].

As is well-known, type II BPS D-branes couple to Ramond-Ramond gauge fields through the Wess-Zumino action [4] (see [5] for string computations checking the form of this action):

$$S_{\rm WZ} = \frac{T_p}{\kappa} \int_{p+1} C \wedge {\rm Tr}\, {\rm e}^{2\pi\alpha'\, F+B} \wedge \sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}}\,. \tag{1}$$

Here T_p/κ denotes the Dp-brane tension, C a formal sum of R-R potentials, F the gauge field on the brane and B the NS-NS two-form. The trace is over the Chan-Paton indices. Further, R_T and R_N are the curvatures of the tangent and normal bundles of the D-brane world-volume, and $\hat{A}$ denotes the A-roof genus. In the setup of ref. [2], where one starts with an unstable configuration of supersymmetric branes and anti-branes, the coupling on the BPS-brane that remains after tachyon condensation is inherited from the similar coupling of the parent branes. In the scenario of ref. [3], one starts from non-BPS branes, which can decay into a lower BPS brane via tachyon condensation. In this case, it has not been clear how the resulting objects acquire the desired couplings in eq. (1). In this note we argue that all type II non-BPS branes couple universally to Ramond-Ramond fields as given by

$$S'_{\rm WZ} = a \int_{p+1} C \wedge d\, {\rm Tr}\, T\, {\rm e}^{2\pi\alpha'\, F+B} \wedge \sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}}\,, \tag{2}$$

where T is the real, adjoint tachyon field living on the non-BPS brane and a is a constant.[1] One term of this action (the one describing the coupling of a non-BPS Dp-brane to C_p) was discussed in refs. [1, 3]. Below we will show how, upon tachyon condensation, these non-BPS "Wess-Zumino" couplings induce the appropriate Wess-

[1] This constant will be fixed in the next paragraph by imposing that the BPS D$(p-1)$-brane we find there have the expected R-R charge. Then this action predicts the R-R charges of the lower BPS D-branes that can be constructed from the non-BPS Dp-brane.

JHEP06(1999)033

Zumino action for the resulting BPS-branes. The cases D$p \to$ D$(p-1)$ and D$p \to$ D$(p-3)$ will be treated in detail. It will turn out, for instance, that the R-R charges of the D8-branes and D6-branes one constructs from unstable D9-branes [3] have the expected ratio. Moreover, we check the presence of these R-R couplings by performing various disc amplitudes with an open string tachyon inserted at the boundary.

Relation to Wess-Zumino action In ref. [3] Horava described how to construct BPS D$(p-2k-1)$-branes as bound states of (sufficiently many) unstable Dp-branes. The lower-dimensional BPS branes arise as the result of the condensation of a tachyon field into a vortex configuration, accompanied by non-trivial gauge fields. We indicate now how the R-R couplings that we propose in eq. (2) account for the R-R couplings (1) that the stable lower-dimensional brane emerging from the condensation must possess.

Consider first a single non-BPS Dp-brane. There is a real tachyon field living on its worldvolume. The tachyon potential is assumed to be such that the vacuum manifold consists of the two points $\{T_0, -T_0\}$.[2] The tachyon can condense to a non-trivial (anti)-kink configuration $T(x)$ depending on a single coordinate. The R-R coupling (2) on the Dp-brane reads in this case

$$a \int_{p+1} C \wedge dT \wedge \mathrm{e}^{2\pi\alpha' F + B} \wedge \sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}}\,, \tag{3}$$

the first term of which was suggested in ref. [3] and shown to be present by a disc computation (in an alternative formalism) in ref. [1]. It involves the topological density $\partial_x T(x)$, which is localized at the core of the kink and is such that $\int dT(x) = \pm 2T_0$. In the limit of zero size we would have $dT(x) = 2T_0 \delta(x - x_0) dx$, and the above action would take the form[3] of the usual Wess-Zumino effective action for a BPS D$(p-1)$-brane, localized in the x-direction at x_0:

$$2T_0 a \int_p C \wedge \mathrm{e}^{2\pi\alpha' F + B} \wedge \sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}}\,. \tag{4}$$

In reality, the D$(p-1)$-brane will have a certain thickness in the direction of the kink.

[2] The symbol T_0 should not be confused with the tension of a D0-brane, which will never explicitly appear in this paper.

[3] Actually, trying to follow the reduction of a Dp-brane to a lower-dimensional one, there is a puzzle concerning the gravitational part $\sqrt{\hat{A}(R_T)/\hat{A}(R_N)}$. The directions along the parent brane transverse to the smaller brane contribute originally to $\hat{A}(R_T)$. It is not clear to us how they are reassigned to the normal bundle in the reduced action. In fact, this problem seems also to be present for the reduction of brane-antibrane pairs to lower-dimensional BPS branes as in ref. [2], where only the standard WZ actions (1) are involved.

Note that the constant a can be fixed in terms of T_0 by equating $2T_0 a$ with the tension T_{p-1}/κ of a BPS D$(p-1)$-brane. This being done, the remainder of this paragraph provides a non-trivial check on our couplings in eq. (2).

As a less trivial example, let us start from two unstable Dp-branes. The tachyon field T, transforming in the adjoint of the U(2) gauge group, can form a non-trivial vortex configuration in co-dimension three. The tachyon potential is assumed to be such that the minima of T have the eigenvalues $(T_0, -T_0)$, so that the vacuum manifold is $\mathcal{V} = \mathrm{U}(2)/(\mathrm{U}(1) \times \mathrm{U}(1)) = S^2$. The possible stable vortex configurations $T(\mathbf{x})$, depending on 3 coordinates x^i transverse to the $(p-2)$-dimensional core of the vortex, are classified by the non-trivial embeddings of the "sphere at infinity" S^2_∞ into the vacuum manifold, namely by $\pi_2(\mathcal{V}) = \mathbf{Z}$.

Apart from the "center of mass" U(1) subgroup we are in the situation of the Georgi-Glashow model, where the tachyon field $T(\mathbf{x}) = T^a(\mathbf{x})\sigma^a$ (σ^a being the Pauli matrices) sits in the adjoint of SU(2), and the vacuum manifold is described by $T^a T^a = T_0^2$. The vortex configuration of winding number one, which is the 't Hooft-Polyakov monopole, is of the form

$$T(\mathbf{x}) = f(r)\sigma_a x^a \,, \tag{5}$$

where r is the radial distance in the three transverse directions, and the prefactor $f(r)$ goes to a constant for $r \to 0$ and approaches T_0/r for $r \to \infty$.

The finite energy requirement implies that $D_i T^a$ vanishes sufficiently fast at infinity, from which it follows that a vortex is accompanied by a non-trivial gauge field; for the case (5) above, the non-trivial part of the SU(2) gauge field has the form

$$\mathcal{A}_i^a(\mathbf{x}) = h(r)\epsilon^a{}_{ij} x^j \,, \tag{6}$$

with $h(r)$ approaching a constant for $r \to 0$, while $h(r) \sim 1/r^2$ at infinity. The field-strength in the unbroken U(1) direction,

$$\mathcal{G}_{ij} = \frac{T^a}{T_0}\mathcal{F}_{ij}^a \,, \tag{7}$$

corresponds to a non-trivial U(1) bundle on the sphere at infinity, i.e. the magnetic charge $g = \int_{S^2_\infty} \mathcal{G}$ is non-zero (and in fact equals the winding number of the vortex in appropriate units). Thus there is a magnetic charge density in the transverse directions, defined by $d\mathcal{G} = \rho(\mathbf{x}) d^3x$, which is concentrated at the core of the vortex solution. In the zero size limit, there would be a point-like magnetic charge at the location of the core: $\rho(\mathbf{x}) = g\, \delta^3(\mathbf{x} - \mathbf{x}^0)$.

The WZ action (2) for the Dp-brane can be rewritten as

$$a \int_{p+1} C \wedge d\,\mathrm{Tr}\{T\, \mathrm{e}^{2\pi\alpha' \mathcal{F}}\} \wedge \mathrm{e}^{2\pi\alpha' \hat{F} + B} \wedge \sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}} \,, \tag{8}$$

where we have split the U(2) field-strength into its SU(2) part $\mathcal{F}$ and its U(1) part $\hat{F}$. Inserting the 't Hooft-Polyakov configuration for the tachyon and the SU(2) gauge field, we see that eq. (7) involves precisely the magnetic monopole field $\mathcal{G} = T^a \mathcal{F}^a$; we get indeed

$$2\pi\alpha' a \int_{p+1} C \wedge d\mathrm{Tr}\{T\,\mathcal{F}\} \wedge \mathrm{e}^{2\pi\alpha'\hat{F}+B} \wedge \sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}} =$$

$$= 2\pi\alpha' a T_0 \int_{p+1} C \wedge \rho(\mathbf{x}) d^3x \wedge \mathrm{e}^{2\pi\alpha'\hat{F}+B} \wedge \sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}}\,. \tag{9}$$

Thus we have a distribution of D$(p-3)$-brane charge localized at the core of the vortex; in particular, in the limit of zero-size core we recover the R-R couplings (2) of a BPS D$(p-3)$-brane that supports the U(1) gauge field $\hat{F}$.

Since the minimal magnetic charge g is 4π in our units, eq. (9) and the remark after eq. (4) lead to the expected ratio $4\pi^2\alpha'$ for the R-R charges of D$(p-3)$- and D$(p-1)$-branes.

The mechanism described above generalizes to the reduction of a non-BPS Dp-brane to a D$(p-2k-1)$-brane via tachyon condensation, described in [3]. In this case, it is convenient to start with 2^k unstable Dp-branes. The configuration of vorticity one for the tachyon field, which sits in the adjoint of U(2^k), is of the form

$$T(\mathbf{x}) = f(r)\,\Gamma_i x^i\,, \tag{10}$$

where r is the radius in the $2k+1$ transverse dimensions x^i, and the Γ-matrices in these dimensions are viewed as $U(2^k)$ elements. Eq. (10) is a direct generalization of the 't Hooft-Polyakov case, eq. (5). Again, the finite energy requirement should imply a non-trivial gauge field configuration, leading to a non-zero generalized magnetic charge $\int_{S^{2k}_\infty} \mathrm{Tr}\{T(\mathcal{F})^k\}$. In such a background, the WZ action (2) contains the factor $d\,\mathrm{Tr}\{T\mathcal{F}^k\} = \rho(\mathbf{x}) d^{2k+1}x$; the (generalized) magnetic charge density ρ is concentrated at the core of the vortex, and in the zero-size limit reduces to a delta-function in the transverse space. Thus we are left with the WZ action for a D$(p-2k-1)$-brane.

String computation To compute the disc scattering amplitudes necessary to check eq. (2), it is convenient to conformally map the disc to the upper half plane and use the "doubling trick" as described, for instance, in ref. [6]. This trick consists in replacing, e.g., $\bar{X}^\mu(\bar{z})$ by $S^\mu{}_\nu X^\nu(\bar{z})$, where $S^\mu{}_\nu$ is diagonal, with entries 1 in the world-volume and -1 in the transverse directions, and then treating the fields depending on $\bar{z}$ as if $\bar{z}$ were a holomorphic variable living on the lower half plane. The fermionic ψ^μ fields are treated in the same way. As to the spin fields in the R-R sector, for BPS Dp-branes in type IIA $\bar{S}^{\dot{\alpha}}(\bar{z})$ is replaced by $(\gamma^0\gamma^1\cdots\gamma^p)^{\dot{\alpha}}{}_\beta\, S^\beta(\bar{z})$ (where the chirality flips because p is even). For type IIB $\bar{S}^\alpha(\bar{z})$ is replaced by $(\gamma^0\gamma^1\cdots\gamma^p)^\alpha{}_\beta\, S^\beta(\bar{z})$, where now p is odd. For the non-BPS Dp-branes we are studying here, p is odd in

IIA and even in IIB, so that there is a chirality flip in IIB and not in IIA. The explicit computations below will be done for IIA, but the story is, of course, completely analogous for IIB.

The first amplitude we are going to compute is the two point function of one open string tachyon and a R-R potential in the presence of a single non-BPS Dp-brane in IIA.[4] This will establish the first term in the expansion of eq. (2). We take the R-R vertex operator in the $(-1/2, -1/2)$ picture (which exhibits the R-R field strengths rather than the potentials):

$$V_{RR} = H_{\alpha\dot\beta}\, S^\alpha(z)\, \bar S^{\dot\beta}(\bar z)\, e^{\mathrm{i}k\cdot X(z,\bar z)} \to H_{\alpha\dot\beta}\, S^\alpha(z)\, (\gamma^0\gamma^1\cdots\gamma^p)^{\dot\beta}{}_{\dot\gamma}\, S^{\dot\gamma}(\bar z)\, e^{\mathrm{i}k\cdot X(z)}\, e^{\mathrm{i}k\cdot S\cdot X(\bar z)}\,, \tag{11}$$

where $H_{\alpha\dot\beta}$ is the bispinor containing the R-R field strengths and k the momentum of the R-R potential. We have omitted the superghost part and do not keep track of the overall normalization, since we are not able to directly determine the constant a in eq. (2) anyway.[5] The tachyon vertex operator is put in the -1 picture:

$$V_T = T(k')\, e^{\mathrm{i}k'\cdot X(y)}\,, \tag{12}$$

where T and k' are the tachyon polarization and momentum and y is a point on the real axis. Again, the superghost part is not displayed. The three insertion points z, $\bar z$ and y can be fixed by introducing ghost fields. Then the contributions of the ghost, superghost and X sectors combine into $(z-\bar z)^{5/4}$. The contraction of the two spin fields in the fermionic sector gives

$$\langle S^\alpha(z)\, S^{\dot\gamma}(\bar z)\rangle = (z-\bar z)^{-5/4}\, C^{\alpha\dot\gamma}\,, \tag{13}$$

with C the charge conjugation matrix. The amplitude becomes

$$T\, H_{\alpha\dot\beta}(\gamma^0\gamma^1\cdots\gamma^p)^{\dot\beta}{}_{\dot\gamma}\, C^{\alpha\dot\gamma} \times K\,, \tag{14}$$

where K is a global factor. Tracing over the spinor indices, only the part of $H_{\alpha\dot\beta}$ proportional to $H_{\mu_1\ldots\mu_{p+1}}(C\gamma^{\mu_1\cdots\mu_{p+1}})_{\alpha\dot\beta}$ contributes, making the amplitude proportional to $T\, H_{\mu_1\ldots\mu_{p+1}}\epsilon^{\mu_1\cdots\mu_{p+1}}$. Upon integration by parts, this confirms the first term of eq. (2).

There is a kinematical subtlety in this computation. String scattering amplitudes can only be computed for on-shell external particles. It is easy to convince oneself that, since the tachyon carries only momentum along the brane and the momentum along the brane is conserved, the tachyon and the R-R potential cannot be both

[4] This has been done before in a formalism in which non-BPS D-branes are constructed in an alternative way [1].

[5] However, we will be interested in the relative normalization of this amplitude with respect to the ones with photons inserted. The constant a itself was fixed in the previous paragraph.

JHEP06(1999)033

on-shell. As a way out, one could consider branes with euclidean signature, for which this kinematical problem does not occur, and then extrapolate the couplings one finds there to their Minkowski cousins.

To check the second term of eq. (2), depending linearly on F, we add to the previous amplitude a vertex operator for a gauge field. This vertex operator is in the 0 picture:

$$V_A = A_\mu \left(\mathrm{i}\dot{X}^\mu(w) + 2\alpha' p \cdot \psi \psi^\mu(w)\right) e^{\mathrm{i}p\cdot X(w)}\,, \tag{15}$$

where this time we have kept track of all normalization factors. Here A_μ is the polarization of the gauge field, p is its momentum and w is on the real axis. Only the fermionic part of the photon vertex operator can lead to terms of the type we are looking for (the photon should provide two gamma-matrices), so we ignore possible contributions from the bosonic part. We will compute the amplitude to lowest order in the photon momentum. This means that we will put p equal to zero in the bosonic sector, thus keeping only the explicit p dotted with a ψ in eq. (15). We follow the previous computation as closely as possible by fixing again z, $\bar{z}$ and y, such that only w needs to be integrated over. In the limit of small photon momentum the ghost, superghost and X sector contributions are unchanged (they multiply to $(z-\bar{z})^{5/4}$). The fermionic correlator is

$$2\alpha' p_\nu A_\mu \langle S^\alpha(z)\, \psi^\nu \psi^\mu(w)\, S^{\dot{\gamma}}(\bar{z})\rangle = -\mathrm{i}\,\alpha' p_\nu A_\mu\, (\gamma^{\nu\mu})^{\alpha\dot{\gamma}}\, (w-z)^{-1}(w-\bar{z})^{-1}(z-\bar{z})^{-1/4}\,. \tag{16}$$

The resulting integral can be done by a contour integration:

$$(z-\bar{z}) \int_{-\infty}^{+\infty} dw\, (w-z)^{-1}(w-\bar{z})^{-1} = 2\pi\mathrm{i}\,, \tag{17}$$

leading to

$$2\pi\alpha'\, p_\nu\, A_\mu\, T\, H_{\alpha\dot{\beta}}\, (\gamma^{\nu\mu})^{\alpha\dot{\gamma}} (\gamma^0\gamma^1\cdots\gamma^p)^{\dot{\beta}}{}_{\dot{\gamma}} \times K \tag{18}$$

for the amplitude. This corresponds indeed to the term in eq. (2) linear in F. Note that the factor $2\pi\alpha'$ multiplying F in eq. (2) comes out correctly.

The generalization to multiple (low momentum) photon insertions is straightforward. The dependence on the photon insertion points of the relevant part of the fermionic correlator factorizes, such that each integration reduces to the one-dimensional integral described in the previous paragraph. It is also easy to include Chan-Paton factors in the above computations, leading to the trace in eq. (2). Finally, one could check the presence of the gravitational terms in eq. (2) explicitly. Since all graviton vertex operators can be inserted in the $(0,0)$ picture, the various contractions will be identical to the ones used in ref. [5].

Note that, from a technical point of view, the only role of the tachyon in the above computations is to provide its superghost part, allowing one to insert the R-R vertex operator in the $(-1/2,-1/2)$ picture, instead of the $(-3/2,-1/2)$ one. Thus

JHEP06(1999)033

the inclusion of the tachyon proves wrong one's first impression that non-BPS D-branes cannot couple to the closed string R-R sector because of the GSO-projection. Apart from this, the above computations perfectly parallel their counterparts for BPS D-branes.

Acknowledgement We would like to thank Pieter-Jan De Smet and Walter Troost for discussions.

References

[1] A. Sen, *J. High Energy Phys.* **09** (1998) 023 [hep-th/9808141]; *J. High Energy Phys.* **12** (1998) 021, [hep-th/9812031]; *Non-BPS states and branes in string theory*, hep-th/9904207;
A. Lerda and R. Russo, *Stable non-BPS states in string theory: a pedagogical review*, hep-th/9905006.

[2] E. Witten, *J. High Energy Phys.* **12** (1998) 019 [hep-th/9810188].

[3] P. Horava, *Adv. Theor. Math. Phys.* **2** (1998) 1373 [hep-th/9812135].

[4] M. Green, J. Harvey and G. Moore, *Class. and Quant. Grav.* **14** (1997) 47 [hep-th/9605033];
Y.-K. Cheung and Z. Yin, *Nucl. Phys.* **B 517** (1998) 69 [hep-th/9710206];
R. Minasian and G. Moore, *J. High Energy Phys.* **11** (1997) 002 [hep-th/9710230].

[5] B. Craps and F. Roose, *Phys. Lett.* **B 445** (1998) 150 [hep-th/9808074]; *Phys. Lett.* **B 450** (1999) 358, [hep-th/9812149];
J. Morales, C. Scrucca and M. Serone, *Nucl. Phys.* **B 552** (1999) 291 [hep-th/9812071];
B. Stefanski, *Nucl. Phys.* **B 548** (1999) 275 [hep-th/9812088].

[6] A. Hashimoto and I. Klebanov, *Nucl. Phys.* **55B** *(Proc. Suppl.)* (1997) 118 [hep-th/9611214].

JHEP06(1999)033